Multiplying and Dividing Fractions

ESSENTIAL QUESTION
How can you use products and quotients of fractions to solve real-world problems?

Real-World Video
To find your average rate of speed, divide the distance you traveled by the time you traveled. If you ride in a taxi and drive \( \frac{3}{2} \) mile in \( \frac{1}{4} \) hour, your rate was 2 mi/h which may mean you were in heavy traffic.

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LESSON 3.1
Multiplying Fractions
6.3.B, 6.3.E

LESSON 3.2
Multiplying Mixed Numbers
6.3.B, 6.3.E

LESSON 3.3
Dividing Fractions
6.3.A, 6.3.E

LESSON 3.4
Dividing Mixed Numbers
6.3.A, 6.3.E
Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.

**Write an Improper Fraction as a Mixed Number**

**EXAMPLE** \( \frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} \)

Write as a sum using names for one plus a proper fraction.

Write each name for one as one.

Add the ones.

Write the mixed number.

Write each improper fraction as a mixed number.

1. \( \frac{9}{4} \)
2. \( \frac{8}{3} \)
3. \( \frac{23}{6} \)
4. \( \frac{11}{2} \)
5. \( \frac{17}{5} \)
6. \( \frac{15}{8} \)
7. \( \frac{33}{10} \)
8. \( \frac{29}{12} \)

**Multiplication Facts**

**EXAMPLE** \( 7 \times 6 = \)

Use a related fact you know.

\( 6 \times 6 = 36 \)

Think: \( 7 \times 6 = (6 \times 6) + 6 \)

\( = 36 + 6 \)

\( = 42 \)

\( 7 \times 6 = 42 \)

Multiply.

9. \( 6 \times 5 \)
10. \( 8 \times 9 \)
11. \( 10 \times 11 \)
12. \( 7 \times 8 \)
13. \( 9 \times 7 \)
14. \( 8 \times 6 \)
15. \( 9 \times 11 \)
16. \( 11 \times 12 \)

**Division Facts**

**EXAMPLE** \( 63 \div 7 = \)

Think: \( 7 \) times what number equals 63?

\( 7 \times 9 = 63 \)

\( 63 \div 7 = 9 \)

So, \( 63 \div 7 = 9 \).

Divide.

17. \( 35 \div 7 \)
18. \( 56 \div 8 \)
19. \( 28 \div 7 \)
20. \( 48 \div 8 \)
21. \( 36 \div 4 \)
22. \( 45 \div 9 \)
23. \( 72 \div 8 \)
24. \( 40 \div 5 \)
**Active Reading**

**Layered Book** Before beginning the module, create a layered book to help you learn the concepts in this module. Label each flap with lesson titles. As you study each lesson, listen and take notes on important ideas, such as vocabulary and formulas, under the appropriate flap. Refer to your finished layered book as you work on exercises from this module.

**Visualize Vocabulary**

Use the ✔ words to complete the triangle. Write the review word that fits the description in each section of the triangle.

- **part of a whole**
- **top number of fraction**
- **bottom number of fraction**

**Understand Vocabulary**

In each grouping, select the choice that is described by the given vocabulary word.

1. **reciprocals**
   - A 1:15
   - B \( \frac{3}{4} \div \frac{1}{6} \)
   - C \( \frac{3}{5} \) and \( \frac{5}{3} \)

2. **mixed number**
   - A \( \frac{1}{3} - \frac{1}{5} \)
   - B \( 3\frac{1}{2} \)
   - C \(-5\)

3. **whole number**
   - A \(-1\)
   - B \(7\)
   - C \(\frac{2}{5}\)

**Vocabulary**

**Review Words**
- area (área)
- denominator (denominador)
- factor (factor)
- fraction (fracción)
- length (longitud)
- mixed number (número mixto)
- numerator (numerator)
- product (producto)
- width (ancho)

**Preview Words**
- reciprocals (recíprocos)
- model (modelo)
- whole number (número entero)
Unpacking the TEKS
Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

**What It Means to You**

You will learn how to divide two fractions. You will also understand the relationship between multiplication and division.

**UNPACKING EXAMPLE 6.3.A**

Zachary is making vegetable soup. The recipe makes $6\frac{3}{4}$ cups of soup. How many $1\frac{1}{2}$-cup servings will the recipe make?

\[
6\frac{3}{4} \div 1\frac{1}{2} = \frac{27}{4} \div \frac{3}{2} = \frac{27}{4} \cdot \frac{2}{3} = \frac{9}{2} = 4\frac{1}{2}
\]

The recipe will make $4\frac{1}{2}$ servings.

**What It Means to You**

You will learn how to determine whether the product of a number and a fraction will be greater than or less than the number.

**UNPACKING EXAMPLE 6.3.B**

Will the product of $\frac{6}{10} \times 6$ be less than 6 or greater than 6?

The product will be less than 6 because $\frac{6}{10}$ is less than 1.

\[
\frac{6}{10} \times 6 = \frac{36}{10} = 3\frac{6}{10}
\]
EXPLORE ACTIVITY  

Modeling Fraction Multiplication

Sam and Pete had a party. After the party, they discovered that \( \frac{3}{4} \) of a casserole was left over. Sam and Pete ate \( \frac{1}{2} \) of the leftover casserole. What fraction of the original casserole did Sam and Pete eat?

Shade the model to show \( \frac{1}{2} \times \frac{3}{4} \).

A. Shade the rectangle to represent the \( \frac{3}{4} \) of the casserole that was left over after the party.

B. Double shade \( \frac{1}{2} \) of \( \frac{3}{4} \). Divide the remaining fourth into two parts so that all of the parts are equal.

C. Sam and Pete ate ____ of the original casserole.

D. Did the amount of casserole increase or decrease when multiplied by \( \frac{1}{2} \)? How does the model show this? Explain.

E. Write the multiplication shown by the model. \( \frac{1}{2} \times \frac{3}{4} = \frac{\_\_\_}{\_\_\_} \)

Reflect

1. **Communicate Mathematical Ideas** Will the product of \( \frac{1}{2} \) and \( \frac{2}{3} \) be greater or less than \( \frac{2}{3} \)? Explain.
Multiplying Fractions

To multiply two fractions you first multiply the numerators and then multiply the denominators. Write the product in simplest form.

\[
\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}} = \frac{\text{numerator}}{\text{denominator}}
\]

EXAMPLE 1

Multiply. Write the product in simplest form.

**A** \[ \frac{1}{3} \times \frac{3}{5} \]

\[
\frac{1}{3} \times \frac{3}{5} = \frac{1 \times 3}{3 \times 5}
\]

Write the problem as a single fraction.

\[
= \frac{3}{15}
\]

Multiply numerators. Multiply denominators.

\[
= \frac{3 \div 3}{15 \div 3}
\]

Simplify by dividing by the GCF.

\[
= \frac{1}{5}
\]

The GCF of 3 and 15 is 3.

Write the answer in simplest form.

**B** \[ \frac{6}{7} \times \frac{2}{3} \]

\[
\frac{6}{7} \times \frac{2}{3} = \frac{6 \times 2}{7 \times 3}
\]

Write the problem as a single fraction.

\[
= \frac{12}{21}
\]

Simplify before multiplying using the GCF.

\[
= \frac{2 \times 2}{7 \times 1}
\]

Multiply numerators. Multiply denominators.

\[
= \frac{4}{7}
\]

Reflect

2. **Communicate Mathematical Ideas** Is the product less than or greater than the factors? Explain.

3. **Analyze Relationships** How can you determine when to simplify using the GCF before multiplying?

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Multiply. Write each product in simplest form.

4. \( \frac{1}{6} \times \frac{3}{5} \)

5. \( \frac{3}{4} \times \frac{7}{9} \)

6. \( \frac{3}{7} \times \frac{2}{3} \)

7. \( \frac{4}{5} \times \frac{2}{7} \)

Multiplying Fractions and Whole Numbers

To multiply a fraction by a whole number, you rewrite the whole number as a fraction and multiply the two fractions.

**EXAMPLE 2**

A class has 18 students. The teacher asks how many students in the class have pets and finds \( \frac{5}{9} \) of the students have pets. How many students have pets?

**STEP 1**

Estimate the product. Multiply the whole number by the nearest benchmark fraction. \( \frac{5}{9} \) is close to \( \frac{1}{2} \), so multiply \( \frac{1}{2} \) times 18.

\[
\frac{1}{2} \times 18 = 9
\]

**STEP 2**

Multiply. Write the product in simplest form.

\[
\frac{5}{9} \times 18 = \frac{5 \times 18}{9}
\]

Rewrite 18 as a fraction.

\[
\frac{5}{9} \times \frac{18}{1}
\]

Simplify before multiplying using the GCF.

\[
\frac{5 \times 2}{9 \times 1}
\]

Multiply numerators. Multiply denominators.

\[
\frac{10}{1} = 10
\]

Simplify by writing as a whole number.

10 students have pets.

**Reflect**

8. Analyze Relationships Is the product of a fraction and a whole number greater than or less than the whole number? Explain.
Guided Practice

1. Lisa, Taryn, and Catherine go to a store to buy party supplies. The store has a sale on the supplies they want for \( \frac{3}{4} \) the original price. The girls agree to each pay \( \frac{1}{3} \) of the cost. (Explore Activity)
   
a. Draw a model to show what fraction of the original price they will each pay.
   
b. What fraction of the original price did each girl pay? __________________
   
c. Write the multiplication shown by the model. __________________
   
d. Did the fraction representing the sale price increase or decrease when multiplied by \( \frac{1}{3} \)? Explain.
      __________________
      __________________

Multiply. Write each product in simplest form. (Example 1)

2. \( \frac{1}{2} \times \frac{5}{8} \)  
3. \( \frac{3}{5} \times \frac{5}{9} \)  
4. \( \frac{3}{8} \times \frac{2}{5} \)

Find each amount. (Example 2)

5. \( \frac{1}{4} \) of 12 bottles of water = _____ bottles  
6. \( \frac{2}{3} \) of 24 bananas = _____ bananas  
7. \( \frac{3}{5} \) of $40 restaurant bill = $ _____  
8. \( \frac{5}{6} \) of 18 pencils = _____ pencils

ESSENTIAL QUESTION CHECK-IN

9. How can you multiply two fractions?
      ________________________________________________
      ________________________________________________
3.1 Independent Practice

Solve. Write each answer in simplest form.

10. Erin buys a bag of peanuts that weighs $\frac{3}{4}$ of a pound. Later that week, the bag is $\frac{2}{3}$ full. How much does the bag of peanuts weigh now? Show your work.

11. **Multistep** Marianne buys 16 bags of potting soil that comes in $\frac{5}{8}$-pound bags.
   
   a. How many pounds of potting does Marianne buy?
   
   b. If Marianne's father calls and says he needs 13 pounds of potting soil, will 4 more bags be enough to cover the extra soil needed?

12. **Analyze Relationships** Name three different pairs of fractions that have the same product when multiplied. Explain how you found them.

13. Marcial found a recipe for fruit salad that he wanted to try to make for his birthday party. He decided to triple the recipe.
   
   a. What is the new amount for the oranges, apples, blueberries, and peaches?

**Fruit Salad**
- $3\frac{1}{2}$ cups thinly sliced rhubarb
- 15 seedless grapes, halved
- $\frac{1}{2}$ orange, sectioned
- 10 fresh strawberries, halved
- $\frac{3}{5}$ apple, cored and diced
- $\frac{2}{3}$ peach, sliced
- 1 plum, pitted and sliced
- $\frac{1}{4}$ cup fresh blueberries

b. **Communicate Mathematical Ideas** The amount of rhubarb in the original recipe is $3\frac{1}{2}$ cups. Using what you know of whole numbers and what you know of fractions, explain how you could triple that mixed number.
14. **Music** Two-fifths of the instruments in the marching band are brass. One-eighth of the brass instruments are tubas.
   
a. What fraction of the band is tubas? ________________________________
   
b. If there are 240 band instruments total, how many are tubas? ________________________________

15. Compare simplifying before multiplying fractions with simplifying after multiplying the fractions.

   _____________________________________________________________

   _____________________________________________________________

16. **Sports** Kevin is a quarterback on the football team. He completed 36 passes during the season. His second-string replacement, Mark, completed \( \frac{2}{9} \) as many passes as Kevin. How many passes did Mark complete?

   _____________________________________________________________

H.O.T. **Focus on Higher Order Thinking**

17. **Represent Real-World Problems** Kate wants to buy a new bicycle from a sporting goods store. The bicycle she wants normally sells for $360. The store has a sale where all bicycles cost \( \frac{5}{6} \) of the regular price. What is the sale price of the bicycle?

   _____________________________________________________________

18. **Error Analysis** To find the product \( \frac{3}{7} \times \frac{4}{9} \), Cameron simplified \( \frac{3}{7} \) to \( \frac{1}{7} \) and then multiplied the fractions \( \frac{1}{7} \) and \( \frac{4}{9} \) to find the product \( \frac{4}{63} \). What is Cameron's error?

   _____________________________________________________________

   _____________________________________________________________

   _____________________________________________________________

19. **Justify Reasoning** When multiplying a whole number by a fraction, the whole number is written as a fraction by placing the value of the whole number in the numerator and 1 in the denominator. Does this change the final answer? Explain why or why not.

   _____________________________________________________________

   _____________________________________________________________

   _____________________________________________________________
EXPLORE ACTIVITY

Modeling Mixed-Number Multiplication

Your town is building a new playground. One third of the $1\frac{3}{4}$-acre playground will be developed to include a swimming pool and splash pad. Find the number of acres that will be used for the swimming pool and splash pad.

Follow these steps to model the situation and find $\frac{1}{3} \times \ \square \ \square$.

A. Shade the model to show the amount of land taken up by the entire playground.

B. Double shade $\frac{1}{3}$ of $1\frac{3}{4}$ to show the portion of the land that is taken up by the swimming pool and splash pad. Divide the remaining fourth into three parts so that all of the parts are equal.

C. ______ acre will be used for the swimming pool and splash pad.

D. Did the fraction representing the entire playground increase or decrease when multiplied by $\frac{1}{3}$? Explain.

Reflect

1. Make a Conjecture  Will the product $1\frac{1}{2} \times \frac{1}{2}$ be greater than or less than $\frac{1}{2}$? Explain.
Multiplying Fractions and Mixed Numbers

To rename a mixed number as a fraction, first multiply the denominator of the fraction by the whole number. Then add the product to the numerator.

**EXAMPLE 1**

The science club members are planting a garden. They used \(2 \frac{1}{4}\) bags of gravel as a border for the flower section of the garden. They will need \(\frac{1}{3}\) as much gravel for the vegetable section as for the flower section. How much gravel will they use in the vegetable section?

1. **STEP 1**
   - Estimate the product. Round the mixed number to the nearest whole number. Find the nearest benchmark for the fraction.
   - \(2 \frac{1}{4}\) is close to 2, so multiply \(\frac{1}{3}\) times 2.
   - \(\frac{1}{3} \times 2 = \frac{2}{3}\)

2. **STEP 2**
   - Multiply. Write the product in simplest form.
   - \(\frac{1}{3} \times 2 \frac{1}{4} = \frac{1}{3} \times \frac{9}{4}\)
   - \(= \frac{1 \times 9}{3 \times 4} = \frac{3}{4}\)
   - The science club members will need \(\frac{3}{4}\) bag of gravel for the vegetable section of the garden.

**YOUR TURN**

Multiply. Write each product in simplest form.

2. \(\frac{3}{3} \times \frac{3}{4}\)
3. \(\frac{4}{5} \times \frac{1}{2}\)
4. \(\frac{5}{6} \times 2 \frac{3}{4}\)
5. \(\frac{3}{5} \times 2 \frac{1}{5}\)
6. \(\frac{9}{10} \times 4 \frac{1}{3}\)
7. \(\frac{5}{6} \times \frac{1}{8}\)
Multiplying Mixed Numbers

Multiplying a mixed number by another mixed number is the same as multiplying a fraction by a mixed number. Rewrite each mixed number as a fraction. Then multiply the numerators and multiply the denominators.

**EXAMPLE 2**

Grace is making $2\frac{1}{2}$ batches of muffins for her school’s annual bake sale. If one batch of muffins requires $1\frac{1}{4}$ cups of flour, how many cups of flour does Grace need to make $2\frac{1}{2}$ batches of muffins?

**STEP 1** Estimate the product. Round each mixed number to the nearest whole number.

$2\frac{1}{2}$ is close to 3 and $1\frac{1}{4}$ is close to 1, so multiply 3 by 1.

$3 \times 1 = 3$

**STEP 2** Multiply. Write the product in simplest form.

$2\frac{1}{2} \times 1\frac{1}{4}$

$2\frac{1}{2} \times 1\frac{1}{4} = \frac{5}{2} \times \frac{5}{4}$

Rewrite each mixed number as a fraction greater than 1.

Multiply numerators.

Multiply denominators.

Write the fraction greater than 1 as a mixed number.

Grace will need $3\frac{1}{8}$ cups of flour.

Reflect

8. **Analyze Relationships** When you multiply two mixed numbers, will the product be less than or greater than the factors? Use an example to explain.

YOUR TURN

Multiply. Write each product in simplest form.

9. $2\frac{2}{3} \times 1\frac{1}{7}$

10. $2\frac{3}{8} \times 1\frac{3}{5}$

11. $4\frac{1}{2} \times 3\frac{3}{7}$

12. $5\frac{1}{4} \times 4\frac{2}{3}$
1. Mr. Martin’s yard is $1\frac{1}{3}$ acres. He wants to plant grass on $\frac{1}{6}$ of his yard. (Explore Activity)
   
a. Draw a model to show how many acres will be covered by grass.

![Model Diagram]

b. How many acres will be covered by grass? ________________

c. Write the multiplication shown by the model. ________________

d. Will the mixed number that represents the original size of Mr. Martin’s yard increase or decrease when multiplied by $\frac{1}{6}$? Explain. ________________

Multiply. Write each product in simplest form. (Example 1 and Example 2)

2. $1\frac{1}{5} \times \frac{3}{5} = $ ________________

3. $1\frac{3}{4} \times \frac{4}{7} = $ ________________

4. $1\frac{5}{6} \times \frac{2}{5} = $ ________________

5. $1\frac{7}{10} \times \frac{4}{5} = $ ________________

6. $\frac{5}{9} \times 3\frac{9}{10} = $ ________________

7. $\frac{7}{8} \times 3\frac{1}{3} = $ ________________

8. $2\frac{1}{5} \times 2\frac{3}{5} = $ ________________

9. $4\frac{3}{4} \times 3\frac{4}{5} = $ ________________

10. How can you multiply two mixed numbers?

   ________________
   ________________
   ________________
   ________________
   ________________

ESSENTIAL QUESTION CHECK-IN
Estimate. Then solve.

11. Carly is making $3\frac{1}{2}$ batches of biscuits. If one batch calls for $2\frac{1}{3}$ cups of flour, how much flour will she need?

12. Bashir collected $4\frac{1}{3}$ baskets of peaches at an orchard. If each basket holds 21 peaches, how many peaches did he collect in all?

13. Jared used $1\frac{2}{5}$ bags of soil for his garden. He is digging another garden that will need $\frac{1}{2}$ as much soil as the original. How much will he use total?

14. Critical Thinking Is the product of two mixed numbers less than, between, or greater than the two factors? Explain.

15. There are approximately $402\frac{1}{4}$ meters around a typical running track. Sandra has challenged herself to run 10 laps a day for 5 days. How many meters will Sandra run if she meets her challenge?

16. Ron wants to make a rectangular basketball court. What is the area of Ron's court?

17. Each of 15 students will give a $1\frac{1}{2}$-minute speech in English class.
   a. How long will it take to give the speeches?
   b. If the teacher begins recording on a digital camera with an hour available, is there enough time to record everyone if she gives a 15-minute introduction at the beginning of class and every student takes a minute to get ready? Explain.
   c. How much time is left on the digital camera?
18. Communicate Mathematical Ideas  How is multiplying a whole number by a mixed number the same as multiplying two mixed numbers?

19. Critique Reasoning  To find the product $3\frac{3}{8} \times 4\frac{1}{9}$, Tara rewrote $3\frac{3}{8}$ as $\frac{25}{8}$ and $4\frac{1}{9}$ as $\frac{37}{9}$. Then she multiplied the fractions to find the product $\frac{221}{72}$. What were her errors?

20. Represent Real-World Problems  Ian is making his special barbecue sauce for a party. His recipe makes $3\frac{1}{2}$ cups of barbecue sauce and uses $2\frac{1}{4}$ tablespoons of soy sauce. He wants to increase his recipe to make five times as much barbecue sauce. He checks his refrigerator and finds that he has 8 tablespoons of soy sauce. Will he have enough soy sauce? Explain.

21. Analyze Relationships  Is it possible to find the product of two mixed numbers by multiplying the whole number parts together, then multiplying the two fractional parts together, and finally adding the two products? Use an example to support your answer.
How do you divide fractions?

**EXPLORE ACTIVITY 1**

**Modeling Fraction Division**

In some division problems, you may know a number of groups and need to find how many or how much are in each group. In other division problems, you may know how many there are in each group, and need to find the number of groups.

A. You have \( \frac{3}{4} \) cup of salsa for making burritos. Each burrito requires \( \frac{1}{8} \) cup of salsa. How many burritos can you make?

To find the number of burritos that can be made, you need to determine how many \( \frac{1}{8} \)-cup servings are in \( \frac{3}{4} \) cups. Use the diagram. How many eighths are there in \( \frac{3}{4} \)? ______

You have enough salsa to make ______ burritos.

B. Five people share \( \frac{1}{2} \) pound of cheese equally. How much cheese does each person receive?

To find how much cheese each person receives, you need to determine how much is in each of ______ parts.

How much is in each part? ______

Each person will receive ______ pound.

**Reflect**

1. Write the division shown by each model.

   _________________________________
Reciprocals

Another way to divide fractions is to use reciprocals. Two numbers whose product is 1 are reciprocals.

\[
\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1
\]

\[
\frac{3}{4} \quad \text{and} \quad \frac{4}{3}
\]

are reciprocals.

To find the reciprocal of a fraction, switch the numerator and denominator.

\[
\text{numerator} \quad \frac{\text{denominator}}{\text{numerator}} = 1
\]

Example 1

Find the reciprocal of each number.

A. \(\frac{2}{9}\)  
   \[
   \frac{2}{9} \quad \frac{9}{2}
   \]
   The reciprocal of \(\frac{2}{9}\) is \(\frac{9}{2}\).

B. \(\frac{1}{8}\)  
   \[
   \frac{8}{1}
   \]
   The reciprocal of \(\frac{1}{8}\) is \(\frac{8}{1}\) or 8.

C. 5  
   \[
   \frac{5}{1}
   \]
   Rewrite as a fraction.
   \[
   \frac{5}{\frac{1}{5}}
   \]
   Switch the numerator and the denominator.
   The reciprocal of 5 is \(\frac{1}{5}\).

Reflect

2. Is any number its own reciprocal? If so, what number(s)? Justify your answer.

___________________________________________________________

3. Communicate Mathematical Ideas Does every number have a reciprocal? Explain.

___________________________________________________________

___________________________________________________________

4. The reciprocal of a whole number is a fraction with ________ in the numerator.

YOUR TURN

Find the reciprocal of each number.

5. \(\frac{7}{8}\)_________________  

6. 9_________________  

7. \(\frac{1}{11}\)_________________
EXPLOR ACTIVITY 2

Using Reciprocals to Find Equivalent Values

A Complete the table below.

<table>
<thead>
<tr>
<th>Division</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 ÷ 2</td>
<td>6 × 7 ÷ 2 =</td>
</tr>
<tr>
<td>5 ÷ 3</td>
<td>5 × 8 ÷ 3 =</td>
</tr>
<tr>
<td>1 ÷ 5</td>
<td>1 × 6 ÷ 5 =</td>
</tr>
<tr>
<td>1 ÷ 3</td>
<td>1 × 3 ÷ 1 =</td>
</tr>
</tbody>
</table>

B How does each multiplication problem compare to its corresponding division problem?

C How does the answer to each multiplication problem compare to the answer to its corresponding division problem?

Reflect

8. Make a Conjecture  Use the pattern in the table to make a conjecture about how you can use multiplication to divide one fraction by another.

9. Write a division problem and a corresponding multiplication problem like those in the table. Assuming your conjecture in 8 is correct, what is the answer to your division problem?
Divide $\frac{5}{9} \div \frac{2}{3}$. Write the quotient in simplest form.

**STEP 1** Rewrite as multiplication, using the reciprocal of the divisor.

\[
\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \times \frac{3}{2}
\]

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

**STEP 2** Multiply and simplify.

\[
\frac{5}{9} \times \frac{3}{2} = \frac{15}{18}
\]

Multiply the numerators. Multiply the denominators.

\[
\frac{5}{9} \div \frac{2}{3} = \frac{5}{6}
\]

Write the answer in simplest form.

\[
\frac{15}{3} \div \frac{18}{3} = \frac{5}{6}
\]

**YOUR TURN**

Divide.

10. \[\frac{9}{10} \div \frac{2}{5} = \]

11. \[\frac{9}{10} \div \frac{3}{5} = \]

**Guided Practice**

Find the reciprocal of each fraction. (Example 1)

1. \[\frac{2}{5} \]
2. \[\frac{1}{9} \]
3. \[\frac{10}{3} \]

Divide. (Explore 1, Explore 2, and Example 2)

4. \[\frac{4}{3} \div \frac{5}{3} = \]
5. \[\frac{3}{10} \div \frac{4}{5} = \]
6. \[\frac{1}{2} \div \frac{2}{5} = \]

**ESSENTIAL QUESTION CHECK-IN**

7. How do you divide fractions?

______________________________
8. Alison has \( \frac{1}{2} \) cup of yogurt for making fruit parfaits. Each parfait requires \( \frac{1}{8} \) cup of yogurt. How many parfaits can she make?

9. A team of runners is needed to run a \( \frac{1}{4} \)-mile relay race. If each runner must run \( \frac{1}{16} \) mile, how many runners will be needed?

10. Trevor paints \( \frac{1}{6} \) of the fence surrounding his farm each day. How many days will it take him to paint \( \frac{3}{4} \) of the fence?

11. Six people share \( \frac{3}{5} \) pound of peanuts equally. What fraction of a pound of peanuts does each person receive?

12. **Biology** If one honeybee makes \( \frac{1}{12} \) teaspoon of honey during its lifetime, how many honeybees are needed to make \( \frac{1}{2} \) teaspoon of honey?

13. Jackson wants to divide a \( \frac{3}{4} \)-pound box of trail mix into small bags. Each of the bags will hold \( \frac{1}{12} \) pound of trail mix. How many bags of trail mix can Jackson fill?

14. A pitcher contains \( \frac{2}{3} \) quart of lemonade. If an equal amount of lemonade is poured into each of 6 glasses, how much lemonade will each glass contain?

15. How many tenths are there in \( \frac{4}{5} \)?

16. You make a large bowl of salad to share with your friends. Your brother eats \( \frac{1}{3} \) of it before they come over.
   a. You want to divide the leftover salad evenly among six friends. What expression describes the situation? Explain.
   b. What fractional portion of the original bowl of salad does each friend receive?
17. **Interpret the Answer** The length of a ribbon is \( \frac{3}{4} \) meter. Sun Yi needs pieces measuring \( \frac{1}{3} \) meter for an art project. What is the greatest number of pieces measuring \( \frac{1}{3} \) meter that can be cut from the ribbon? How much ribbon will be left after Sun Yi cuts the ribbon? Explain your reasoning.

__________________________________________________________________

__________________________________________________________________

18. **Represent Real-World Problems** Liam has \( \frac{9}{10} \) gallon of paint for painting the birdhouses he sells at the craft fair. Each birdhouse requires \( \frac{1}{20} \) gallon of paint. How many birdhouses can Liam paint? Show your work.

__________________________________________________________________

__________________________________________________________________

19. **Justify Reasoning** When Kaitlin divided a fraction by \( \frac{1}{2} \), the result was a mixed number. Was the original fraction less than or greater than \( \frac{1}{2} \)? Explain your reasoning.

__________________________________________________________________

__________________________________________________________________

20. **Communicate Mathematical Ideas** The reciprocal of a fraction less than 1 is always a fraction greater than 1. Why is this?

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________

21. **Make a Prediction** Susan divides the fraction \( \frac{5}{8} \) by \( \frac{1}{16} \). Her friend Robyn divides \( \frac{5}{8} \) by \( \frac{1}{32} \). Predict which person will get the greater quotient. Explain and check your prediction.

__________________________________________________________________

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EXPLORE ACTIVITY

Modeling Mixed Number Division

Antoine is making sushi rolls. He has $2\frac{1}{2}$ cups of rice and will use $\frac{1}{4}$ cup of rice for each sushi roll. How many sushi rolls can he make?

A. To find the number of sushi rolls that can be made, you need to determine how many fourths are in $2\frac{1}{2}$. Use fraction pieces to represent $2\frac{1}{2}$ on the model below.

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B. How many fourths are in $2\frac{1}{2}$? ________________

Antoine has enough rice to make ________________ sushi rolls.

Reflect

1. **Communicate Mathematical Ideas** Which mathematical operation could you use to find the number of sushi rolls that Antoine can make? Explain.

2. **Multiple Representations** Write the division shown by the model.

3. **What If?** Suppose Antoine instead uses $\frac{1}{8}$ cup of rice for each sushi roll. How would his model change? How many rolls can he make? Explain.

**Lesson 3.4**

Dividing Mixed Numbers

**ESSENTIAL QUESTION**

How do you divide mixed numbers?

**EXPLORE ACTIVITY**

Modeling Mixed Number Division

Antoine is making sushi rolls. He has $2\frac{1}{2}$ cups of rice and will use $\frac{1}{4}$ cup of rice for each sushi roll. How many sushi rolls can he make?

A. To find the number of sushi rolls that can be made, you need to determine how many fourths are in $2\frac{1}{2}$. Use fraction pieces to represent $2\frac{1}{2}$ on the model below.

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B. How many fourths are in $2\frac{1}{2}$? ________________

Antoine has enough rice to make ________________ sushi rolls.

Reflect

1. **Communicate Mathematical Ideas** Which mathematical operation could you use to find the number of sushi rolls that Antoine can make? Explain.

2. **Multiple Representations** Write the division shown by the model.

3. **What If?** Suppose Antoine instead uses $\frac{1}{8}$ cup of rice for each sushi roll. How would his model change? How many rolls can he make? Explain.

**Lesson 3.4**

Dividing Mixed Numbers

**ESSENTIAL QUESTION**

How do you divide mixed numbers?
Using Reciprocals to Divide Mixed Numbers

Dividing by a fraction is equivalent to multiplying by its reciprocal. You can use this fact to divide mixed numbers. First rewrite the mixed numbers as fractions greater than 1. Then multiply the first fraction by the reciprocal of the second fraction.

**EXAMPLE 1**

One serving of Harold’s favorite cereal contains \(1 \frac{2}{5}\) ounces. How many servings are in a \(17 \frac{1}{2}\)-ounce box?

**STEP 1** Write the situation as a division problem.

\[17 \frac{1}{2} \div 1 \frac{2}{5}\]

You need to find how many groups of \(1 \frac{2}{5}\) are in \(17 \frac{1}{2}\).

**STEP 2** Rewrite the mixed numbers as fractions greater than 1.

\[17 \frac{1}{2} \div 1 \frac{2}{5} = \frac{35}{2} \div \frac{7}{5}\]

**STEP 3** Rewrite the problem as multiplication using the reciprocal of the second fraction.

\[\frac{35}{2} \div \frac{7}{5} = \frac{35}{2} \times \frac{5}{7}\]

The reciprocal of \(\frac{7}{5}\) is \(\frac{5}{7}\).

**STEP 4** Multiply.

\[\frac{35}{2} \times \frac{5}{7} = \frac{5 \times 5}{2 \times 1}\]

Simplify first using the GCF.

\[= \frac{25}{2} = 12 \frac{1}{2}\]

Multiply numerators. Multiply denominators.

Write the result as a mixed number.

There are 12 \(\frac{1}{2}\) servings of cereal in the box.

**Reflect**

4. **Analyze Relationships** Explain how you can check the answer.

5. **What If?** Harold serves himself \(1 \frac{1}{2}\)-ounce servings of cereal each morning. How many servings does he get from a box of his favorite cereal? Show your work.
Solving Problems Involving Area

Recall that to find the area of a rectangle, you multiply length \( \times \) width. If you know the area and only one dimension, you can divide the area by the known dimension to find the other dimension.

**EXAMPLE 2**

The area of a rectangular sandbox is \(56 \frac{2}{3}\) square feet. The length of the sandbox is \(8 \frac{1}{2}\) feet. What is the width?

**STEP 1** Write the situation as a division problem.

\[
56 \frac{2}{3} \div 8 \frac{1}{2}
\]

**STEP 2** Rewrite the mixed numbers as fractions greater than 1.

\[
56 \frac{2}{3} \div 8 \frac{1}{2} = \frac{170}{3} \div \frac{17}{2}
\]

**STEP 3** Rewrite the problem as multiplication using the reciprocal of the second fraction.

\[
\frac{170}{3} \div \frac{17}{2} = \frac{170}{3} \times \frac{2}{17}
\]

**STEP 4** Multiply.

\[
\frac{170}{3} \times \frac{2}{17} = \frac{170 \times 2}{3 \times 17} = \frac{20}{3}, \text{ or } 6 \frac{2}{3}
\]

The width of the sandbox is \(6 \frac{2}{3}\) feet.

**Reflect**

7. **Check for Reasonableness** How can you determine if your answer is reasonable?
YOUR TURN

8. The area of a rectangular patio is $12\frac{3}{8}$ square meters. The width of the patio is $2\frac{3}{4}$ meters. What is the length? ______

9. The area of a rectangular rug is $14\frac{1}{2}$ square yards. The length of the rug is $4\frac{1}{3}$ yards. What is the width? ______

Guided Practice

Divide. Write each answer in simplest form. (Explore Activity and Example 1)

1. \[4\frac{1}{4} ÷ \frac{3}{4} = \]

\[\frac{4}{4} ÷ \frac{3}{4} = \]

\[\frac{4}{4} × \frac{4}{3} = \]

2. \[1\frac{1}{2} ÷ 2\frac{1}{4} = \]

\[\frac{2}{2} ÷ \frac{4}{1} = \]

\[\frac{2}{2} × \frac{1}{4} = \]

3. \[4 ÷ 1\frac{1}{8} = \]

4. \[3\frac{1}{5} ÷ 1\frac{1}{7} = \]

5. \[8\frac{1}{3} ÷ 2\frac{1}{2} = \]

6. \[15\frac{1}{3} ÷ 3\frac{5}{6} = \]

Write each situation as a division problem. Then solve. (Example 2)

7. A sandbox has an area of 26 square feet, and the length is $5\frac{1}{2}$ feet. What is the width of the sandbox?

\[\text{width} = \frac{26}{5\frac{1}{2}} = \]

8. Mr. Webster is buying carpet for an exercise room in his basement. The room will have an area of 230 square feet. The width of the room is $12\frac{1}{2}$ feet. What is the length?

\[\text{length} = \frac{230}{12\frac{1}{2}} = \]

ESSENTIAL QUESTION CHECK-IN

9. How does dividing mixed numbers compare with dividing fractions?

---------------------------------------------------------------------------------------------------------------------------------------
10. Jeremy has \(4\frac{1}{2}\) cups of iced tea. He wants to divide the tea into \(\frac{3}{4}\)-cup servings. Use the model to find the number of servings he can make.

11. A ribbon is \(3\frac{2}{3}\) yards long. Mae needs to cut the ribbon into pieces that are \(\frac{2}{3}\) yard long. Use the model to find the number of pieces she can cut.

12. Dao has \(2\frac{3}{8}\) pounds of hamburger meat. He is making \(\frac{1}{4}\)-pound hamburgers. Does Dao have enough meat to make 10 hamburgers? Explain.

13. **Multistep** Zoey made \(5\frac{1}{2}\) cups of trail mix for a camping trip. She wants to divide the trail mix into \(\frac{3}{4}\)-cup servings.
   
   a. Ten people are going on the camping trip. Can Zoey make enough \(\frac{3}{4}\)-cup servings so that each person on the trip has one serving?

   
   b. What size would the servings need to be for everyone to have a serving? Explain.

   
   c. If Zoey decides to use the \(\frac{3}{4}\)-cup servings, how much more trail mix will she need? Explain.

14. The area of a rectangular picture frame is \(30\frac{1}{3}\) square inches. The length of the frame is \(6\frac{1}{2}\) inches. Find the width of the frame.
15. The area of a rectangular mirror is $11\frac{11}{16}$ square feet. The width of the mirror is $2\frac{3}{4}$ feet. If there is a 5 foot tall space on the wall to hang the mirror, will it fit? Explain.

16. Ramon has a rope that is $25\frac{1}{2}$ feet long. He wants to cut it into 6 pieces that are equal in length. How long will each piece be?

17. Eleanor and Max used two rectangular wooden boards to make a set for the school play. One board was 6 feet long, and the other was $5\frac{1}{2}$ feet long. The two boards had equal widths. The total area of the set was $60\frac{3}{8}$ square feet. What was the width?

18. Draw Conclusions Micah divided $11\frac{2}{3}$ by $2\frac{5}{6}$ and got $4\frac{2}{17}$ for an answer. Does his answer seem reasonable? Explain your thinking. Then check Micah’s answer.

19. Explain the Error To divide $14\frac{2}{3} \div 2\frac{3}{4}$, Erik multiplied $14\frac{2}{3} \times 4\frac{3}{4}$. Explain Erik’s error.

20. Analyze Relationships Explain how you can find the missing number in $3\frac{4}{5} \div \square = 2\frac{5}{7}$. Then find the missing number.
3.1 Multiplying Fractions

Multiply.

1. \( \frac{4}{5} \times \frac{3}{4} \)  
2. \( \frac{5}{7} \times \frac{9}{10} \)

3. Fred had 264 books in his personal library. He donated \( \frac{2}{11} \) of these books to the public library. How many books did he donate?

3.2 Multiplying Mixed Numbers

Multiply.

4. \( \frac{3}{8} \times 2 \frac{1}{2} \)  
5. \( 3 \frac{3}{5} \times \frac{5}{6} \)

6. Jamal and Dorothy were hiking and had a choice between two trails. One was 5 \( \frac{1}{3} \) miles long, and the other was 1 \( \frac{3}{4} \) times as long. How long was the longer trail?

3.3 Dividing Fractions

Divide.

7. \( \frac{7}{8} \div \frac{3}{4} \)  
8. \( \frac{4}{5} \div \frac{6}{7} \)

9. \( \frac{1}{3} \div \frac{7}{9} \)  
10. \( \frac{1}{3} \div \frac{5}{8} \)

3.4 Dividing Mixed Numbers

Divide.

11. \( 3 \frac{1}{3} \div \frac{2}{3} \)  
12. \( 1 \frac{7}{8} \div 2 \frac{2}{5} \)

13. \( 4 \frac{1}{4} \div 4 \frac{1}{2} \)  
14. \( 8 \frac{1}{3} \div 4 \frac{2}{7} \)

ESSENTIAL QUESTION

15. Describe a real-world situation that is modeled by multiplying two fractions or mixed numbers.
Selected Response

1. Which of the following statements is correct?
   A. The product of $\frac{3}{4}$ and $\frac{7}{8}$ is less than $\frac{7}{8}$.
   B. The product of $1\frac{1}{3}$ and $\frac{9}{10}$ is less than $\frac{9}{10}$.
   C. The product of $\frac{3}{4}$ and $\frac{7}{8}$ is greater than $\frac{7}{8}$.
   D. The product of $\frac{7}{8}$ and $\frac{9}{10}$ is greater than $\frac{9}{10}$.

2. Which shows the GCF of 18 and 24 with $\frac{18}{24}$ in simplest form?
   A. GCF: 3; $\frac{3}{4}$
   B. GCF: 3; $\frac{6}{8}$
   C. GCF: 6; $\frac{3}{4}$
   D. GCF: 6; $\frac{6}{8}$

3. A jar contains 133 pennies. A bigger jar contains $1\frac{2}{7}$ times as many pennies. What is the value of the pennies in the bigger jar?
   A. $\$1.49$
   B. $\$1.52$
   C. $\$1.68$
   D. $\$1.71$

4. Which of these is the same as $\frac{3}{5} \div \frac{4}{7}$?
   A. $\frac{3}{5} \div \frac{7}{4}$
   B. $\frac{4}{7} \div \frac{3}{5}$
   C. $\frac{3}{5} \times \frac{4}{7}$
   D. $\frac{3}{5} \times \frac{7}{4}$

5. What is the reciprocal of $3\frac{3}{7}$?
   A. $\frac{7}{24}$
   B. $\frac{3}{7}$
   C. $\frac{7}{3}$
   D. $\frac{24}{7}$

6. A rectangular patio has a length of $12\frac{1}{2}$ feet and an area of $103\frac{1}{8}$ square feet. What is the width of the patio?
   A. $4\frac{1}{8}$ feet
   B. $8\frac{1}{4}$ feet
   C. $16\frac{1}{2}$ feet
   D. 33 feet

Gridded Response

7. Jodi is cutting out pieces of paper that measure $8\frac{1}{2}$ inches by 11 inches from a large sheet that has an area of 1,000 square inches. What is the area of each piece of paper that Jodi is cutting out written as a decimal?